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Network-Theoretic Approach for Analyzing Connectivity in Air Transportation Networks

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Research reported in this paper is motivated by the need to better understand the structure of connectivity in air transportation networks. The aim is to investigate analysis models and techniques from modern network theory as a framework to provide both characterization of network structure and a useful systems analysis approach to derive implications from both local and global topology characteristics. Recent developments in network theory establish means to quantify topological structure in a manner that may indicate expected performance and robustness. In this paper, a mathematical footing from network theory is introduced for examining transport networks in the U.S. domestic air transportation system. Using data for the 2004 travel year, the structure of the transport network (service routes between airports) and several subnetworks is exposed in terms of degree distribution, and the importance of airports is assessed through several network measures. Useful implications are drawn from measures and further analysis that directly maps these measures to system performance is presented. The general approach is found to merit further investigation as part of a larger, more comprehensive, and design-oriented systems analysis framework for air transportation.

Nomenclature

A_{ij} = element of nonweighted adjacency matrix
A_{ij}^w = element of weighted adjacency matrix (linking strength)
B_i = betweenness centrality
C_{avg} = average clustering coefficient
C_i = clustering coefficient of a given node i
E_t = transporting efficiency
$\langle k \rangle$ = average degree
k_i = degree of a given node i
l_{avg} = average shortest path
l_i = average shortest path length between node i and the other nodes
l_{ij} = shortest path length between node i and node j
l_i^w = weighted average shortest path length between node i and the other nodes
N = number of nodes in a network
N_l = number of links in a network
$P(k)$ = cumulative degree distribution
$p(k)$ = degree distribution
s_i = load of a given node i
w_{ij} = demand between node i and node j
x_i = eigenvector centrality
ρ = network density
σ_{st} = number of shortest paths between node s and node t

I. Introduction

THE research questions addressed in this paper concern the structure of air transportation networks. Specifically, our aim is to investigate analysis models and techniques from modern network theory as a framework to provide both insight on the overall structure and a useful systems analysis approach to derive implications of both local and global topology characteristics.

Such system-level tools are important because of the significant challenge posed by seeking to understand and the influence of air transportation networks. The evolution of these networks and the operations on them are a function of a complex web of relationships, only some of which directly involve the aircraft and associated technologies. Further, in the air transportation system (ATS), "no one is completely in charge"; that is, there is no central authority that completely determines how individual participants behave and thus how the overall system and its performance evolve. Instead, the development of future ATS states unfolds via the combined choices made and systems fielded by these participants, for example, service providers, travelers, manufacturers, infrastructure providers, etc. Donohue aptly describes the ATS as a complex adaptive system [1], and we have further characterized it as a system of systems (SoS) [2]. This latter designation highlights the multiple, heterogeneous systems that interact on numerous levels and in various networks but that also operate independently. Interactions that result in "emergent behavior" are hallmarks of complexity and motivate the need to study network structure and evolution.

Therefore, analysis at the ATS level requires a shift from traditional approaches in which systems are designed (and optimized) largely in isolation and then integrated. Systems and services must be understood in terms of their connectivity to enable better design, as evidenced for aircraft design in two recent studies that couch aircraft design in a network context [3,4]. Further, a broader characterization of how air transportation is understood as a system-of-systems problem is presented in [5]. The specific aims of the present paper are to contribute toward this methodological shift by exploring the efficacy of a network-theoretic approach to understand connectivity in the ATS and demonstrate use of associated analyses and resulting implications for operational and policy decisions.

There are multiple, interacting networks embedded in the ATS. One characterization of these networks has been presented by Holmes [6], using the analogy of an open systems interconnection (OSI) stack in communication network design. A modified version of this characterization, adopting a more clearly top-down construct, is presented in Table 1. In light of this characterization, a network-centric abstraction and analysis approach for study of the ATS seems appropriate [7]. One goal for such an approach is to address the fact that some networks employ the same resources but in a different

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Table 1 Characterization of several networks in the ATS

Network	Node (N) & link (L)	Time scale of change
Demand	N: Homes/businesses L: Demand for trips	Months/years
Mobility	N: Origin/destination locations L: Actual passenger trips	Days/Weeks
Transport	N: Airports L: Service routes	Days/Weeks
Operator	N: Aircraft, crew L: Missions	Hours
Infrastructure	N: Waypoints and airports L: Air routes	Months

operational context. For example, nodes in the transport network are airports and the links represent service routes between nodes. The mobility network also contains airports in its nodal structure, in this setting as passenger points of entry into the air transportation system. Consequently, these networks are coupled in a physical manner (e.g., node resource sharing, Fig. 1) and in an operational manner (e.g., overlapping time scales).

Another advantage of network-centric abstraction of the ATS is its applicability at different hierarchical levels. Network analysis can address interactions among networks across these hierarchical levels with unaltered methodology, demonstrating extensibility of the approach. The transport network provides one example of hierarchy. The overall transport network is composed of the scheduled and nonscheduled service subnetworks. The scheduled service subnetwork is composed of the individual airline subnetworks and so on.

Together with the operational and financial policies of organizations that direct them, these networks constitute a high-level model of the ATS. Decision makers could use tools developed from the integrated network model to find network configurations (and enabling components, such as new aircraft and airspace concepts) that lead to desired outcomes for all participants. However, this is an exceedingly difficult task along many dimensions, including 1) the need to know how the structure of each network (its topology) impacts operational performance and 2) the need to understand how each topology varies with time, how it interacts with others, and how the types of processes taking place on the network develop. These issues significantly complicate obtaining stable solutions for global behavior at the system-of-systems level. The fact that some air transportation services in the future will be a traditional, scheduled service while other services may be offered "on demand" exemplifies complications that arise when multiple time scales are at play and integration issues arise between legacy and new systems.

Motivated by this larger set of goals, the research reported in this paper pursues the more focused specific aims stated at the outset by examining the U.S. domestic commercial transport network. The paper is organized as follows. After some key topological measures and models of networks are introduced in Sec. II, an empirical analysis of the 2004 ATS transport network is conducted in Sec. III. Data on the evolution of network structure and mappings to performance metrics are presented and implications described. A framework for network topology design that makes use of such analysis is then proposed in Sec. IV. Finally, key results, conclusions, and implications are summarized in Sec. V.

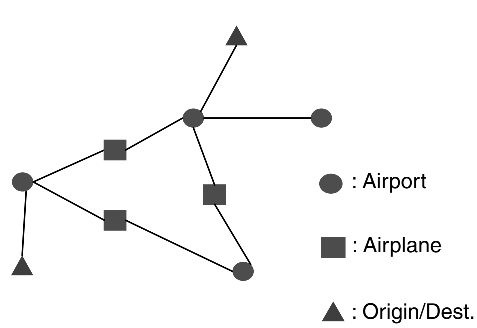


Fig. 1 The mobility and transport networks interact via shared resources and operations.

II. Network Theory: Measures and Models

A. Background on Network Theory

Recent developments in modern network theory (also called "network science") from researchers spanning the physics, mathematics, computer science, informatics, and biology domains have uncovered experimental evidence that real-world networks behave differently than predicted using standard network models. The random network model, one of the classic and widely employed standard network models, was studied extensively by Erdős and Rényi [8]. In random networks, most nodes have the same number of links (termed *degree*), and the distribution of degree across all nodes follows a binominal distribution. However, this model has failed to predict topologies observed in many real networked systems, as documented for a number of cases in a wide array of application domains [9]. As new discoveries concerning the structure and growth of network topologies are being made, it is becoming clear that the robustness and efficiency of a network are highly related to its topology. These advancements are typified by the work of Albert and Barabási [10] in the exploration of scale-free networks, an alternate model to the random network characterized by its power law degree distribution and successful prediction of the evolution of real-world networks such as the Internet (network of routers) and World Wide Web (WWW) (network of web sites). The most notable finding is that, while scale-free networks provide efficient connectivity and greater robustness to random disruptions than a random network, they are also more vulnerable to targeted attacks. However, some more recent studies have opposed this view. For example, Doyle et al. [11] concluded that the Internet, when considered with actual technological and economic considerations in addition to topology, is not scale free but instead an example of a highly optimized tolerance (HOT) network. Such opposed findings are, perhaps, a positive sign of the early development of the field. The situation does give pause in the study of the ATS, calling to attention the context-dependent meaning of topologies and their characteristics.

A small group of researchers has explored its application for analyzing air transportation networks. Guimera et al. [12] analyzed the worldwide air transportation network topology and computed measures which characterized the relative importance of cities/airports. Research within the aeronautics community in this context is just germinating, with one general exploratory study by Kincaid [13] and an analysis of the basic routing structure of the various mainline commercial airlines by Berry et al. [14]. The work of Conway [15] and Holmes [6] recognizes and advocates that network science warrants examination for fundamental studies of air transportation. Bonnefoy and Hansman [16] used a plot of the weighted degree distribution for light jet operations to understand the capability of airports to attract the use of very light jets (VLJs). We have also described preliminary analysis of transport network data and use in simulation [17].

A significant body of works exists in the related domain of operations research on the design of optimal networks for particular instances and applications (e.g., schedule for an airline). However, these approaches generally do not pursue insight into the underlying structures of networks, the role this structure plays in future designs, nor the interplay between networks from multiple domains. One of the notable contributions is that of Lederer and Nambimadom [18] who used a restricted airline network design problem in an operations research-based approach to explore the cost effectiveness of four basic network topologies for a profit maximizing airline. Their findings indicate that, if distance between cities is very small or the number of cities is small, then direct service is optimal. Alternately, hub-and-spoke networks are superior for greater distance and/or large numbers of cities.

However, the examination of ATS using the network theory at the national level and assessment of associated analysis models and techniques as a framework to provide both insight on ATS structure and a useful systems analysis has been largely missing. This constitutes the primary aim of this paper.

B. Statistical Measures of Network Topology

Network theory stands on the foundation of graph theory. A network (or graph) G is a collection of nodes and links for which a particular configuration defines the topology $[G(N, L)]$. The topology is represented via the adjacency matrix A, which is size $N \times N$ for a network with N nodes. Several types of links can exist between nodes, including weighted, unweighted, directed, and undirected. For weighted networks, each link has a corresponding scalar weight that signifies some distinguishing trait (cost, capacity, distance, etc.). In an unweighted network, all links are assumed to have the same weight leading to a binary adjacency matrix [entries indicate simply whether a link between two nodes is present ("1") or not ("0")]. An undirected link imposes a two-way connection between nodes, producing a symmetric adjacency matrix. For directed networks, the links impose a one-way connection between nodes producing a nonsymmetric adjacency matrix. In this paper, only undirected networks are considered and network measures are in accord with this definition.

Once a network topology is recorded in its adjacency matrix, several measures are used to characterize the topology. The most basic measure is the number of links incident to a node, termed its degree k. Other key measures are summarized in the following.

Global measures capture average or aggregate characteristics as follows:

Network density: ratio of number of existing links to number of possible links:

$$\rho = \frac{2N_l}{N(N-1)} \qquad 0 < \rho \leq 1 \tag{1}$$

General implications: Networks with relatively low density are called *sparse* networks. Because higher density implies greater cost and complexity, a design interest is to find sparse networks with good performance.

Degree distribution plots the fraction of nodes with k links; the *cumulative degree distribution* plots the probability that a node has k links or more:

$$\text{degree distribution}: \; p_k$$

$$\text{cumulative dd}: \qquad P(k) = \sum_{k'=k}^{\infty} p_{k'} \tag{2}$$

General implications: Both distributions provide a sense of global structure by characterizing the relative number of nodes by degree. Traits of the distributions can indicate how processes on the network may perform and what types of vulnerability may be important.

The average shortest path length represents the average separation distance between nodes (e.g., the geodesic distance, which is the minimum number of links needed to traverse from one node to another),

$$l_{\text{avg}} = \frac{1}{\frac{1}{2}N(N-1)} \sum_{i \geq j} l_{ij} \tag{3}$$

where $l_{ij} =$ the length of the shortest path between nodes i and j.
General implications: The implication is context dependent. A low l_{avg} is desired from a passenger connectivity perspective. However, such "short connections" may also accelerate the transmission of delay after disruption; for the latter, a higher average, shortest path could slow these effects.

Local measures target distinguishing information about individual nodes and their near neighbors as follows:

Strength, or weight, of a node is the total load on all its links:

$$s_i = \sum_j A_{ij}^w \tag{4}$$

General implications: One indicator of nodal importance; in the transport network, the amount of traffic (operations) associated with the node.

Eigenvector centrality, introduced by Newman [19], used for weighted networks, is proportional to the linear combination of its neighbors' degree and strength of links:

$$x_i = \lambda^{-1} \sum_j A_{ij}^w x_j \tag{5}$$

In a compact form, Eq. (5) becomes $A^w x = \lambda x$, an eigenvector problem where x is an eigenvector of the adjacency matrix.
General implications: In the transport network, the importance of one airport is determined not only by its own number of routes supported, but also the number of routes and traffic level of airports with which it directly connects (i.e., an airport with high eigenvector centrality is likely to be very busy itself and also connected to other busy airports).

Betweenness centrality measure introduced by Freeman [20] assesses nodal importance by counting the fraction of the shortest path through a given node:

$$B_i = \frac{2}{(n-1)(n-2)} \sum_{s \neq t \neq i} \frac{\sigma_{st}(i)}{\sigma_{st}} \tag{6}$$

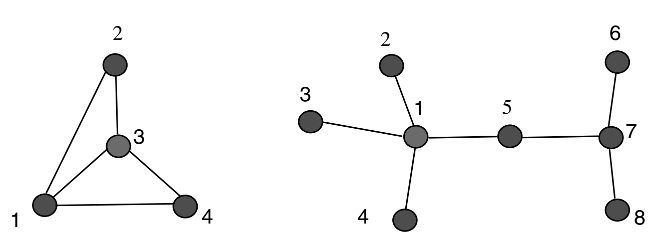
σ_{st} is the number of shortest paths going from node s to node t and $\sigma_{st}(i)$ is the number of shortest paths from node s to node t through a node i.
General implications: Another measure of importance; an otherwise low degree node may be important because it provides connections between separated parts of the network (determined by the number of shortest paths passing through that node).

The clustering coefficient for a given node is the number of triangles centered on that node divided by the number of triples centered on that node:

$$C_i = \frac{1}{k_i(k_i - 1)} \sum_{j,k} A_{ij} A_{ik} A_{jk} \tag{7}$$

The average clustering coefficient C_{avg} is the mean value of all the clustering coefficients of all the nodes in a network.
General implications: The measure of local cohesiveness for a collection of nodes; a node with higher $C_i > C_{\text{avg}}$ is "more interconnected" than the average. This measure has implications on local robustness (or global for the average value). Higher C_i indicates greater robustness because alternate connection paths may exist when a neighboring node fails.

Calculations for two sample networks (Fig. 2) illustrate these measures. For the first network shown in Fig. 2 $k_1 = k_3 = 3$, while $k_2 = k_4 = 2$. The respective clustering coefficients of each node are $C_1 = C_3 = 2/3$ and $C_2 = C_4 = 1$ and the average of the network is $5/6$. The average shortest path is $l_{\text{avg}} = 7/6$. Sample network 2 highlights cases of high betweenness centrality. In this network $B_1 = 0.714$ and $B_5 = 0.571$, whereas the value for all other nodes is zero. The observation from this sample case is that relatively high betweenness occurs for high degree nodes with low connectivity among its adjacent nodes and for nodes which bridge two clusters (node 5 in this case).



Sample Network (1) Sample Network (2)

Fig. 2 Two sample networks.

C. Network Topology Models

The network theory literature has placed a primary emphasis on the study of particular network models for understanding behavior in real-world networks, especially on depicting the topology through degree distributions. Two of the most important network models developed so far, the random network and scale-free network models briefly introduced earlier, are described in more detail in this section.

In the random network model proposed by Erdős and Rényi [8], the nodal degree k follows a binominal distribution in an N-node random graph with connecting probability p:

$$P(k) = \binom{N-1}{k} p^k (1-p)^{N-1-k} \tag{8}$$

When the network size N is sufficiently large, the *binomial distribution* is approximated by a *Poisson distribution* given by

$$p(k) \sim e^{-pN} \frac{(pN)^k}{k!} = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!} \tag{9}$$

The parameter $\langle k \rangle$ represents the expected value of the nodal degree, and it depends on the connecting probability and network size. In random networks, all the nodes are assumed to be identical such that connecting probability remains constant between each pair of nodes. Through such a construction process, a homogeneous network topology is formed; there is a characteristic scale appearing in its degree distribution.

Another well-known network model is the scale-free network model, and the particular model for growth proposed by Barabási and Albert (the so-called BA model) that leads to scale-free topologies [10]. Two mechanisms are included in the BA model: the addition of new nodes and preferential attachment to generate the power law degree distribution that typifies a scale-free network [Eq. (10)]. Preferential attachment means that new nodes added to the system are more likely to link with existing nodes possessing a high degree:

$$p(k) = ak^{-\gamma} \tag{10}$$

The exponent γ and coefficient a theoretically depend on growth rate and types of preferential attachment. In scale-free networks, most of the nodes have few links while few of the nodes have many links, forming the heterogeneous network topology. In air transportation communities, it is common to refer to such networks as having a "hub-and-spoke" topology, though as we shall see these are not necessarily equivalent.

Comparisons of the degree distributions of random and scale-free networks are shown in Fig. 3. Owing to its Poisson distribution, a random network has a symmetric distribution characterized by the existence of peak probability occurring at a particular value of k, as indicated in Fig. 3a. Degree distribution for a scale-free network, on the other hand, lacks such a peak, or scale. Even visual inspection of these two network models makes apparent practical implications. For example, because scale-free networks rely on a few hubs to provide their significant connectivity, it is highly vulnerable for failure of those hubs (whereas the random network has no hubs). However, the scale-free network is moderately more robust to a random node failure (than a random network), because most of the nodes have a low degree.

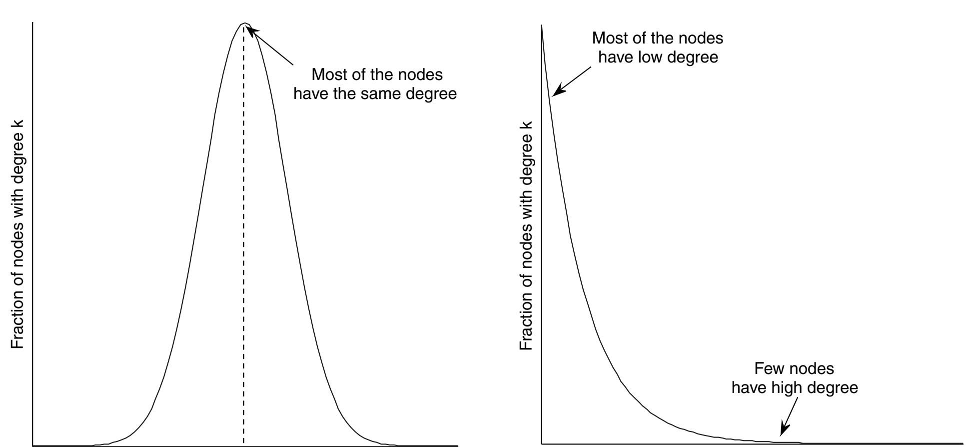
The network measures introduced earlier also clarify differences in network models. In a random network, typically, the value for the average shortest path (l_{avg}) tends to be high and the average clustering coefficient (C_{avg}) is (relatively) low. On the other hand, a high value of C_{avg} can also be achieved in an *ordered lattice* (a network in which each node is connected uniformly to a fixed number of neighbors, and thus, all nodes have the same degree) but at the price of higher l_{avg}. Based upon the seminal work of Watts and Strogatz [21], a class of networks that falls between these was identified and found to occur frequently in the real world. These networks were called "small world" because they enabled seemingly inordinately high connectivity with relatively low density by establishing a few "shortcuts" by rewiring links beginning from a regular lattice. More specifically, small-world networks are characterized by a high clustering coefficient and low shortest path length. The small-world property allows for fast traversing over the network and represents a balance between overall connectivity and the ability for local tailoring. Scale-free networks indeed have the small-world property, but with typically far fewer links and lower network density than a regular lattice.

III. Empirical Analysis of 2004 U.S. Air Transportation Transport Network

In this section, results from an empirical study of the 2004 U.S. domestic air transportation transport network using network theory are presented. The choice of the U.S. domestic network was made to complement recent studies of the global and North American transport networks [12] and to lay a foundation for a new approach for systems analysis at the ATS level.

A. Data Definition and Description

The data are obtained from the Air Carrier Statistics database family maintained by the U.S. Bureau of Transportation Statistics (BTS) [22]. In particular, the T-100 Domestic Segment (all U.S. Carriers) database was used to construct the networks studied. The database contains domestic nonstop segment data reported by U.S. air carriers, including carrier, origin, destination, aircraft type and service class for transported passengers, and other data. Several



a) Random network degree distribution **b) Scale-free network degree distribution**

Fig. 3 Notional degree distributions.

Table 2 Global statistical measures of the 2004 U.S. domestic transport network

	Overall	Scheduled	Nonscheduled
Average clustering coefficient, C_{avg}	0.568	0.618	0.473
Average shortest path, l_{avg}	3.20	3.32	3.385
Network density, ρ	0.024	0.025	0.022
Average degree, $\langle k \rangle$	25.4	21.25	18.66
Network size, N	1054.0	848.0	848.0

different measures are available for use in defining a link, such as the number of flights scheduled, number of flights performed, number of passengers or number of available seats, etc. In this paper, the number of performed passenger flights per year between airports is selected as the entry for each node pair in the overall transport network adjacency matrix. If at least one such flight is performed annually between airports, a link is signified. Rare instances of differences in the directional traffic flow between node pairs were observed. To simplify the network analysis and remain in conformity with the assumption of an undirected network, the value of inbound and outbound traffic is simply averaged in these rare cases. Overall, the network formed from the data consists of 1054 nodes (airports) and 13385 links (service routes).

B. Global Statistical Properties

The global statistical properties for the overall 2004 U.S. air transport network as well as the scheduled and nonscheduled subnetworks are displayed in Table 2. Nonscheduled service is delineated separately in the database and defined as "Revenue flights, such as charter flights, that are not operated in regular scheduled service [22]." Fractional, "air taxi," and general aviation operations are not included in either. The scheduled network, consisting of airline subnetworks, has higher average clustering coefficient, density, and average degree than the unscheduled, implying a network with tighter local connectivity among nodes. Although both subnetworks happen to have the same size, the total operations (not shown) are between 1 and 2 orders of magnitude lower in the nonscheduled. The overall (combined) network creates more pairs of direct flight routes and thus reduces the average shortest path, though some of these paths may be impractical due to lack of collaborative operations between most service providers. Thus, one use of these global connectivity measures is a "best case" assessment and perhaps benchmark for policy/investment studies.

Further insight on structure is embedded in plots of degree distributions, shown in Fig. 4 in linear scale for the overall, scheduled, and nonscheduled networks. It is readily observed in the linear plots that a random network model is not well suited in any case. The degree distribution in log–log is shown on the left in Fig. 5, indicating potential applicability of the scale-free network model but hindered by the scattering at a high degree. To better understand the implications of these distinctions, a notional degree distribution is presented on the right in Fig. 5. Because a scale-free network follows a power law per Eq. (10), its degree distribution should appear linear across at least two decades in each axis in a log–log plot, exemplified by the solid straight line in the notional graph. If the network grows over time via node and link addition, this line shifts away from the origin (dashed line) and highlights the fact that the plot can be used to compute the overall capacity of the network. Looking more carefully at the shape, if at a high degree such as k_2, the fraction of nodes has a lower value of logarithmic $p(k)$ (say p_4), this may indicate that the number of nodes capable of achieving the high degree are less than the power law prediction (sublinear behavior, as found in the VLJ study [16]). There is a capability constraint on high traffic nodes. This rationale is adopted in some network theory literature, termed "truncation tail" or "cutoff." Similarly, at low degree a deficiency of nodes (e.g., p_2 vs p_1 corresponding to k_1 in Fig. 6) can also be evidence of deviation from the scale-free network model.

The search for a power law match in Fig. 5 is complicated by significant scattering in the tail in the logarithmic scale, mainly due to the insufficiently large network size (which inhibits good statistics in the tail). Some researchers use instead plots of the cumulative degree distribution (CDD) to look for signs of power law behavior. The cumulative version reduces the scatter in the tail, but to the detriment of the analysis in that it no longer plots the actual distribution of degree [9]. Displayed in Fig. 6 is the cumulative degree distribution for the 2004 overall network. Numerous analytical models can be explored in search of the best fit. We investigated a number of models, obtaining a reasonable fit for these data with both a sum of two exponentials and a power law effect modified by a constant. Models that combine power law and exponential behavior are also possible. However, when seeking to fit a power law to the CDD using the analytical form of Eq. (10), some amount of artificial cutoff effect is introduced at high degree [17]. Thus, although we can identify capacity constraints from these data, we advise caution to quantify them with high assurance solely from these plots, especially when multiple models provide good fit.

1. Synopsis of Findings/Implications from Global Properties of Aggregate Networks

Cognizant that scattering at a high degree in the degree distribution and a mix of effects in the tail of the cumulative distribution limit the precision in their use, we conclude that the overall, scheduled, and nonscheduled transport networks fail to agree completely with the prediction of a scale-free network model. The deviation at low degree likely implies the lack of "isolated spoke" airports and deviation at high degree results, in part, from capacity deficiency at busy airports.

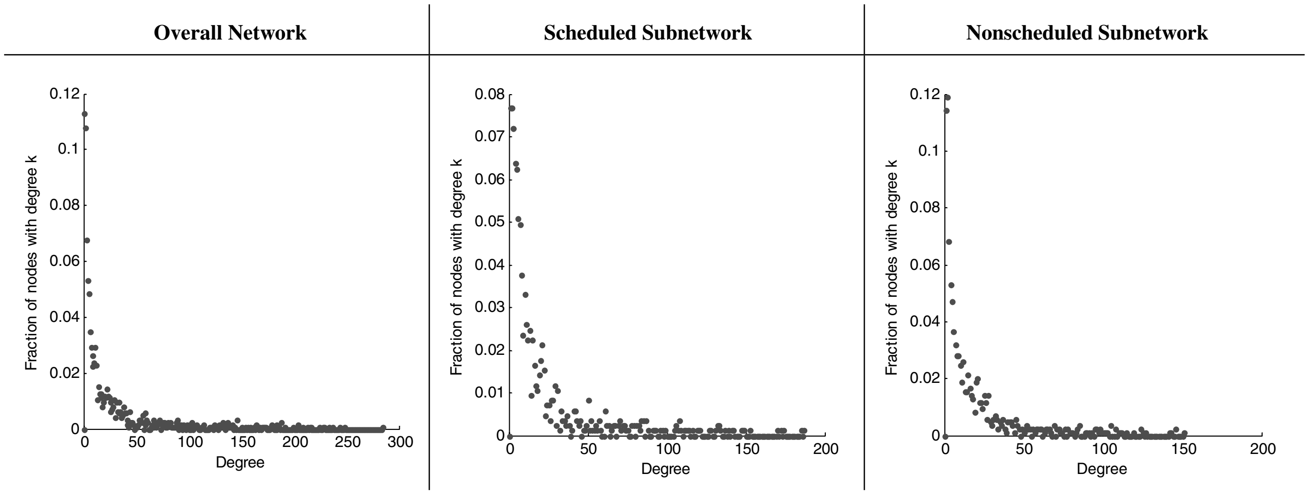


Fig. 4 Degree distributions of transport networks in linear scales.

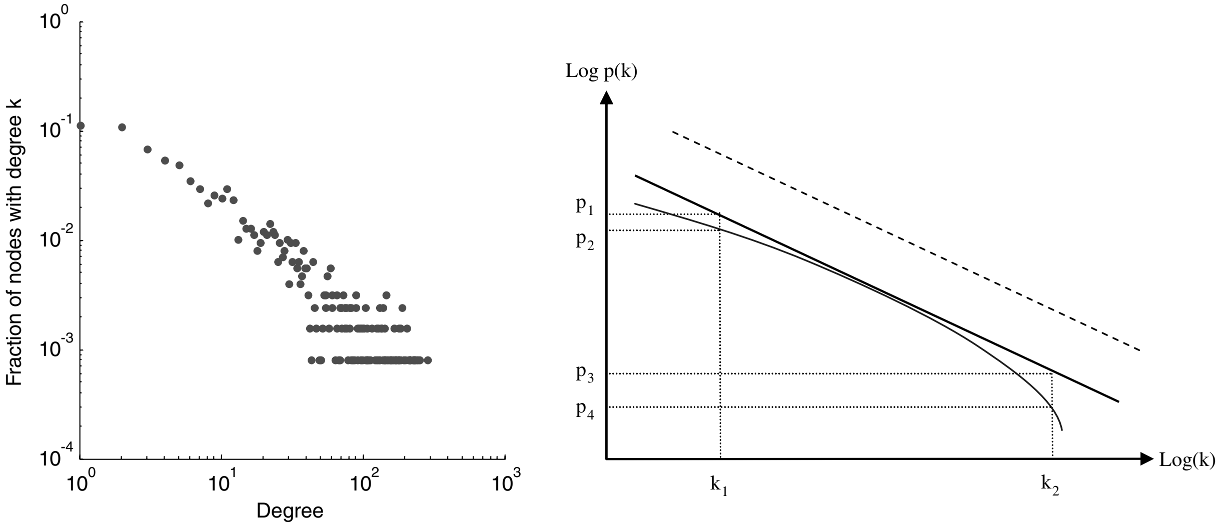


Fig. 5 Degree distribution for overall 2004 transport network in log–log (left) and notional degree distribution in logarithmic scale (right).

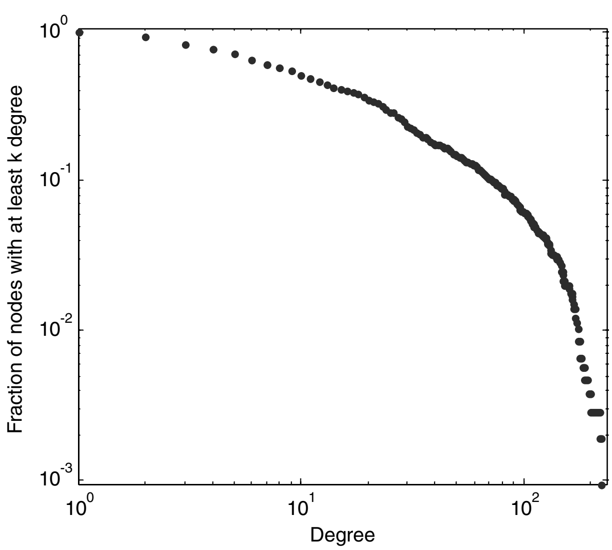


Fig. 6 Cumulative degree distribution for overall 2004 transport network.

Another interpretation is simply that these networks are better represented by a combination of power law behavior with an exponential distribution dominating at a high degree. Finally, despite the appearance of similar degree distributions, the underlying evolutionary processes for scheduled and nonscheduled services are different. The scheduled network is composed of carriers known to employ hub-and-spoke operations while service providers in the nonscheduled network, operating independently and with a higher rate of change in topology, likely execute more of a point-to-point set of operations. The merging on nonscheduled operations together produces an aggregate network of similar traits.

This last observation points to a key *methodological* implication: knowledge of specific subnetwork structures that drive the structure of aggregate topologies is important for understanding network dynamics. This observation is also emerging in the study of natural and other artificial networks, pointing to the importance of addressing modular structure [23]. For example, in the BA model, a power law degree distribution results from the addition of new nodes and node-based preferential attachment both of which are necessary conditions for the formation of a scale-free network. Stimulated by deregulation, the airlines overhauled their route structure, adding both nodes and links in pursuit of cost-efficient operations. However, in recent years the overall transport network topology has evolved primarily from addition, deletion, and rearranging of routes (links), that is, topology reconfiguration [24]. Few airports entered the network as new nodes in the recent years such that the rerouting process dominated the evolution, eliminating one necessary condition in the BA model. In addition, the node-based preferential attachment is not necessarily valid in the context of transport network where the direct flight connection at each pair of airports is largely determined by the demands at both origin and destination airports—pair based preferential attachment. Underlying evolutionary mechanisms in ATS subnetwork networks, therefore, can change with the circumstances and thus so must network models used for analysis.

Further decomposition to lower subnetworks, in this case airlines, is thus in order. Several subnetworks belonging to particular service providers within the scheduled network were investigated. Three examples are summarized across Figs. 7 and 8 and Table 3: Delta Airlines, United Airlines, and Southwest Airlines. While Delta and United's degree distribution indicate their hub-and-spoke strategy (Fig. 7), Southwest's is different and has evolved from 1990 to 2004 (Fig. 8) transforming from a degree distribution resembling a random model to one that, while certainly not scale free, has moved toward a greater use of centralized nodes and a fair number of spokes. A graphical view of Southwest's evolution is displayed in Fig. 9. Small black dots represent existing nodes (airports), large dots represent new nodes added that year, and large gray dots represent nodes deleted that year. The thin lines represent existing links and the thick lines represent new links (unless connected to a large gray node, in which case thick lines represent links deleted in the given year). The evolution possessed both node addition and link creation. Southwest's network growth also illustrates its morphing from a regional carrier to a nationwide carrier. Thus, Southwest is not now (nor ever purely was) a "point-to-point" carrier as is sometimes asserted, though it employs more "secondary" hubs than United and Delta. This latter observation was also pointed out in [14].

Examination of a clustering coefficient over time (Fig. 10a) indicates that, while United, Delta, and American experience a decrease (quite significant in American's case), Southwest shows a marked increase, ending in 2005 near the same value as competitors. A precipitous decline in this metric is seen from 2001 to 2002 for Delta and United, indicating that the disruptive events of 2001 were manifest in a reduction in local cohesiveness which in turn implies less robustness to disruptions at that node in terms of availability of reroutes. Southwest has a similar l_{avg} as competitors (Table 3), though the underlying network structure differs. Southwest achieves this by consistently forming a denser network of links (Fig. 10b) while the other airlines have done so largely by means of further centralization of their connectivity via hubs (their

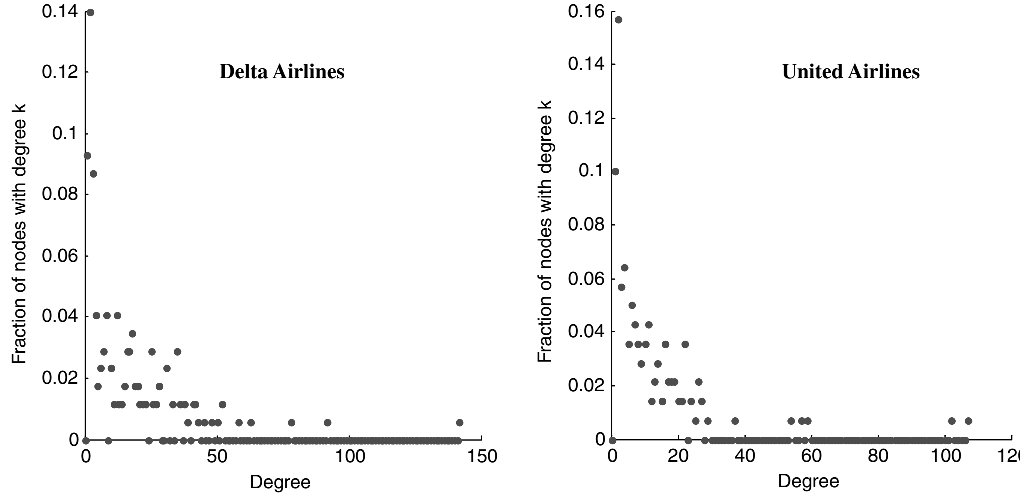


Fig. 7 Degree distribution of flight routes for Delta and United Airlines.

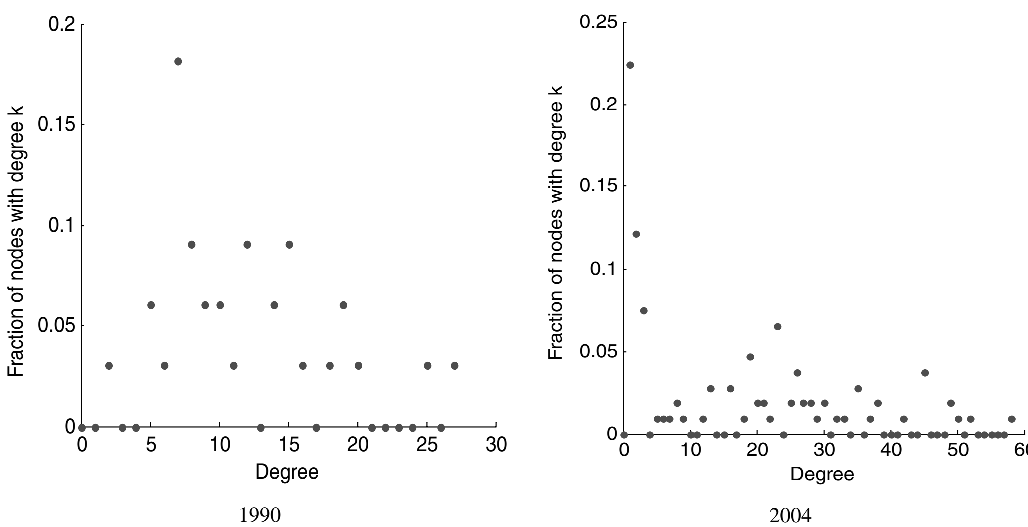


Fig. 8 Degree distributions for Southwest Airlines: 1990 and 2004.

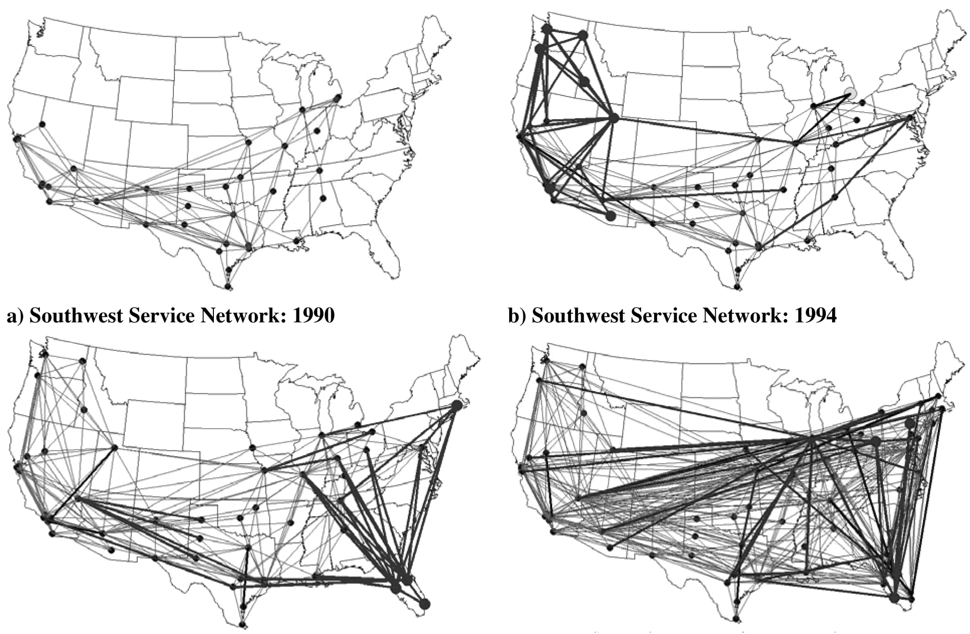
total number of links, not shown, is decreasing but density is remaining fairly constant). The spike in density in 2002 resulted from a combination of real and artificial factors. In that year, the BTS combined reporting from the 298 C database (mostly regional carriers) with the T100, thus increasing significantly the number of nodes (and reducing density) in the ATS-wide network. At the same time, Southwest did indeed significantly increase its route density (in comparison to competitors).

2. Synopsis of Findings/Implications from Global Properties of Airline Subnetworks

Although degree (or cumulative) distributions struggle to convey a definitive picture of the topology on their own (especially with reduced data points in airline subnetworks), they can effectively reflect underlying differences among disparate airlines and changes in network structure when examined over time. Network measures also can serve as efficient models for airline strategic behavior. For example, with a network of greater density but similar average shortest path and clustering coefficient, it could be said that Southwest employs a less efficient network. Yet, Delta and United owe their greatest vulnerability to disruption at hubs (due to their underlying topology) and these hubs, in fact, do often exhibit degraded performance due to weather and capacity constraints. The inefficiency of higher density of Southwest's network is an investment in the ability to overcome the vulnerability to random disruptions. This ability is further enabled by the fact that Southwest employs a homogenous aircraft fleet, providing an example of the linkage between the transport and operator networks. Overall, knowledge of particular airline subnetwork characteristics that

Table 3 Global statistical measures of three major domestic airlines in 2004

Metric	Delta Airlines	United Airlines	Southwest Airlines
Average clustering coefficient, C_{avg}	0.558	0.618	0.540
Average shortest path, l_{avg}	2.1	2.1	2.3
Network density, ρ	0.095	0.085	0.151
Average degree, $\langle k \rangle$	16.2	11.8	15.9
Network size, N	172.0	140.0	107.0



a) Southwest Service Network: 1990 b) Southwest Service Network: 1994

c) Southwest Service Network: 1996 d) Southwest Service Network: 2005

Fig. 9 Evolution of Southwest Airlines service network via topology snapshot visualization.

ultimately drive the structure (and performance) of aggregate topologies seems necessary to develop effective infrastructure and policy enhancements.

C. Local Statistical Properties: Importance (Centrality) of Airports

Although global network properties provide insight into the overall connectivity, the roles of local (i.e., individual nodes and their neighborhood) properties and structure are also important. In the present application, critical airports are identified using the four unique measures introduced in Sec. II. Each measure seeks to capture a different perspective on centrality, or importance to functioning, of the nodes. Results for the overall 2004 U.S. transport network are shown in Table 4. The first column lists the top five airports ranked by strength centrality, which captures the traffic volume operated at each airport. Eigenvector centrality distinguishes airports' importance by considering both the traffic volume at an airport (via its strength) and that of those airports to which it connects. In that case, Chicago O'Hare airport surpasses Atlanta Hartsfield–Jackson due to its housing of two large carriers (United and American) that operate significant interhub traffic. The number of distinct service routes at each airport, its degree, can also represent an airport's importance. Minneapolis–Saint Paul airport (MSP), the largest and busiest airport in the upper Midwestern region and hub of Northwest Airlines, ranked as the most important airport in the ATS in terms of its degree. However, the fact that the degree difference between MSP and other hub airports [e.g., Denver (DEN)] is small and that MSP has relatively low strength centrality (228,731) mitigates its overall importance beyond consideration of degree. [For reference, local statistical measures for the 35 U.S. airports included in the FAA's Operational Evolution Partnership (OEP) are provided in Fig. A1 in the Appendix.]

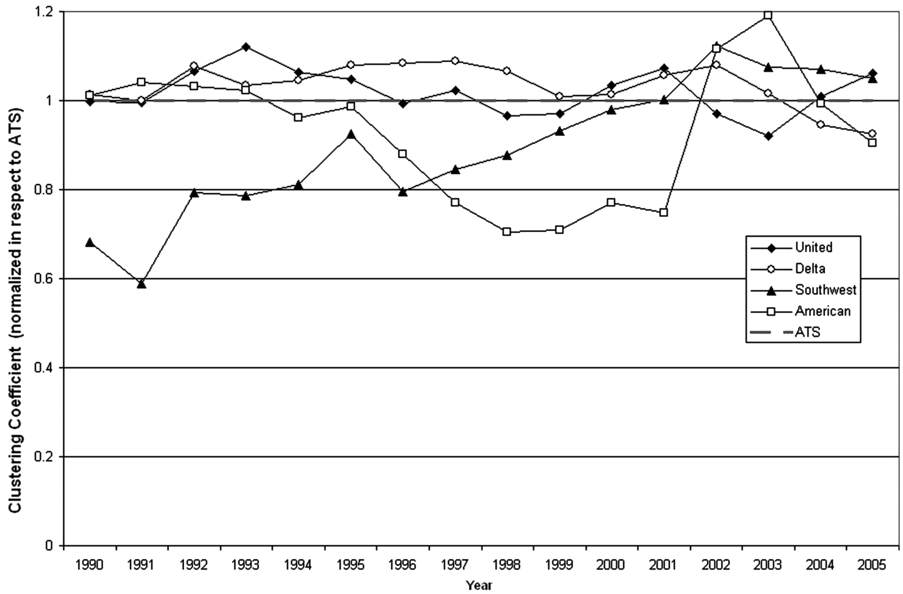
Anchorage airport (ANC) had the highest betweeness centrality value (by a wide margin). In Table 5, the top five airports ranked via betweenness centrality value and their corresponding degree and clustering coefficients are listed. High degree and the low clustering coefficient of ANC indicate low local connectivity in its neighborhood altogether account for its high betweeness centrality value. In practice, ANC serves as a local transfer airport to transport passengers among the many locales in Alaska served by commercial air service. According to BTS data for 2004, 685 airports are located in Alaska comprising about 25% of total airports in the BTS database and resulting in two distinguishable clusters. ANC serves to bridge two major clusters, revealing its crucial role as a transfer airport not just in its intracluster connectivity but also for intercluster transport. Anchorage was also identified by Guimera et al. [12] in their study of the worldwide air transport system. Both findings indicate the importance of Anchorage airport through the lens of betweenness and the connection of isolated clusters.

Implications: The basic questions motivating the analysis of local properties are which nodes are important and why? We find that the answers depend on the different local measures which signify importance in different dimensions. For example, the "biggest" (i.e., busiest) is not necessarily most important through the lens of betweenness centrality, where a high value indicates the importance to maintain a connection between isolated clusters. In a different dimension, maintaining capacity at high strength (and especially high eigenvector centrality) airports is crucial to preventing delay and delay propagation because these airports directly influence so many others. To delve deeper on this point, a multivariate regression model for predicting the airport's number of delayed operations using its degree, clustering coefficient, eigenvector centrality, degree weight, and surrounding population as predictor variables is displayed in Eq. (11). All variables are normalized using the corresponding maximum value, and the model produces a good coefficient of determination ($R^2 = 0.95$). The comparison between the actual and predicted number of delayed operations (for airports that registered at least one delay) is shown in Fig. 11. A 5% error interval is also included. Eigenvector centrality and degree compose the majority of the regression model (significantly high F values and type 2 sum of squares compared to the other variables in the model). The primary methodological implication, then, is that these local measures can be meaningful and efficient indicators of network operational performance,

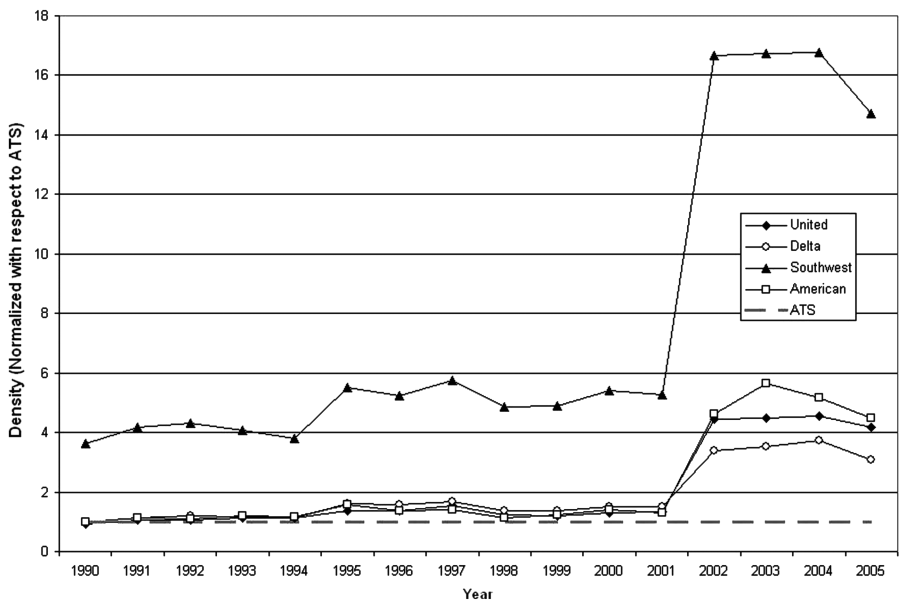
$$\sqrt{\text{delayed ops/year}} = 0.01928 + 0.147^{*}k_i + 0.02606^{*}C_i + 0.56722^{*}x_i + 0.20758^{*}s_i + 0.07462^{*}\text{pop} \tag{11}$$

IV. Future Directions: Toward a Design Capability

The empirical analysis in Sec. III demonstrated that a network-theoretic analysis can generate new and useful knowledge about the structure of connectivity in the ATS transport networks. However, analysis of static topologies alone is insufficient for the design of future connectivity. Further, recalling the "no one is completely in charge" nature of the ATS design must be understood in a context that recognizes the (uncertain) influence of other players. Presently,



a)



b)

Fig. 10 Evolution of C_{avg} and ρ for several airlines.

objective functions useful for optimal connectivity design in a system of systems are not yet well developed nor are evolutionary mechanisms that are appropriate for modeling at higher levels of complexity. In the ATS, objective functions of individual stakeholders differ and levels of authority to allocate resources vary. An airline seeks to design a subnetwork in the overall scheduled network that satisfies perceived demand in a manner that maximizes profit. The Federal Aviation Administration (FAA), in its role as provider of the National Airspace System (NAS) services and regulator, seeks to develop an overall network that maximizes capacity, minimizes delay, and increases reliability. A comprehensive understanding of how evolutionary mechanisms in subnetworks drive the formation of the overall network topology, and proper use of topological measures, is the ultimate research goal but one that requires significant future work.

Toward this end, a design-oriented framework is envisioned that comprises a series of subnetworks and an aggregated network that shares many of the same components of the subnetworks. Data from actual networks, design requirement specifications, and possibly an evolutionary algorithm are used to establish initial network

Table 4 Top five airports by centrality measures in overall transport network

	Strength centrality (average of arrival and departures)		Eigenvector centrality		Degree centrality		Betweenness centrality	
Rank	Value	Airport	Value	Airport	Value	Airport	Value	Airport
1	431,542	ATL	0.3199	ORD	236	MSP	0.2837	ANC
2	426,954	ORD	0.2924	ATL	222	DEN	0.1238	FAI
3	358,568	DFW	0.2612	DFW	221	DFW	0.1125	SEA
4	254,512	DEN	0.2095	LAX	218	ATL	0.0921	MSP
5	242,258	LAX	0.1941	DEN	198	ORD	0.0764	DEN

Table 5 Characteristics of airports that possess the five highest betweenness centrality values

	Betweenness centrality	Degree	Clustering coefficient	Strength centrality (average of arrival and departures)
ANC	0.2837	136	0.1063	42,812
FAI	0.1238	121	0.1183	22,033
SEA	0.1125	146	0.2982	150,970
MSP	0.0921	236	0.2290	228,731
DEN	0.0764	222	0.2181	254,512

topologies. To shape desirable networks, a two-level approach is envisioned in which 1) subnetwork owners would develop a class of local optimal solutions (in the presence of competition), and 2) aggregate level agents would seek to generate the best system-level optimal solution in cognizance of subnetwork actions and pertinent externalities. Subsequent measurement of local and global topology determines performance for the subnetworks. The "best" network topology at any level would depend on particulars of demand requirements, constraints on resources, and expected perturbations or failures, all of which can vary with time; thus, these searches would likely take place as some form of time-integrated (dynamic) optimization.

Scenarios that apply dynamic disturbances could be injected, if desired, and efficiency and robustness are measured. Efficiency is the ability to meet requirements with minimal energy or complexity (e.g., provide needed connectivity with minimal network density). Robustness refers to the resistance of a network to different types of disruptions (such as random failures or targeted attacks). Robustness also implies that efficiency does not degrade in response to variations of system loading (demand). A superior network can sustain different modes of failure and also agilely adapt to fluctuations of system loading without losing its overall performance. Achieving these outcomes may require the aggregate network manager to allocate resources and impose changes on subnetwork constraints. Tension between these various metrics foreshadows a design problem complicated by multiple objectives and limited control of all possible design variables.

V. Conclusions

The research reported in this paper provides an initial examination of the efficacy of a network-theoretic approach to understand aspects of connectivity in the air transportation system, which is composed of several interacting networks. In particular, the transport network, composed of service routes between airports, was selected for detailed study. Local and global measures of connectivity in network topologies derived from the network theory were presented. The important random and scale-free network models were also introduced.

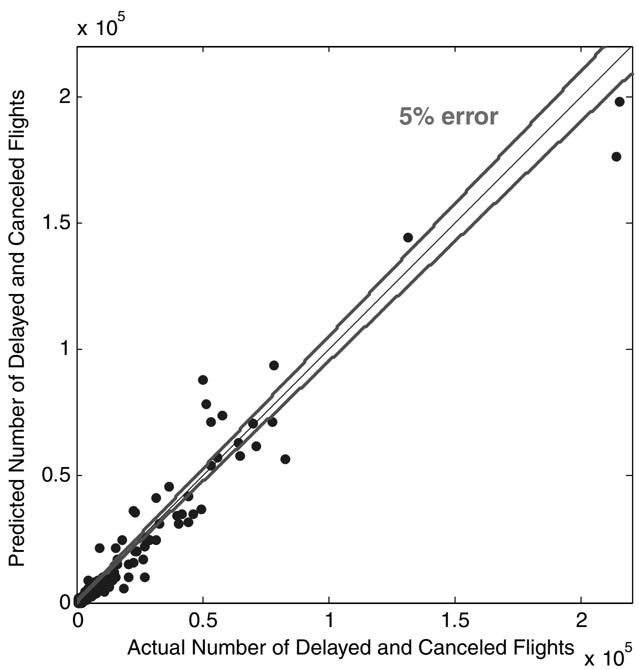


Fig. 11 Prediction validation for delayed operations regression model (each data point is an airport).

Using these models and measures, the U.S. domestic passenger transport network for year 2004 was studied. Several implications are drawn from global network analysis. The degree distributions of the scheduled (i.e., primarily commercial airline), nonscheduled, and overall transport network indicate that these topologies do not resemble a random network model; neither do they conform completely to a scale-free model. Deviation from the power law behavior (indicative of the scale-free model) occurs both at low and high degree, the latter due in part to capacity constraints at high degree airports. The scheduled network had higher local clustering, density, and average degree than the nonscheduled implying tighter local connectivity and robustness among nodes. The overall network (combining nonscheduled and scheduled services) extends the theoretical reach of services on the network, but represents only a best case upper bound on connectivity as present business models limit service provider collaboration in using combined connectivity. These findings point to the need to examine the structure of further decomposed subnetworks and the degree of interaction between subnetworks. As an initial step in this direction, degree distributions for the transport network for three major airlines were studied with results indicating partial correspondence of operational strategy to network models such as the scale-free model. The present degree distribution of Southwest Airlines shows structure that is more distributed than mainline carriers, though not by operating a point-to-point network but indeed employing more secondary hubs. These subnetwork distributions shed greater light on the underlying mechanisms driving the structure in the overall transport network and their further study is recommended, especially within the proposed design framework.

The importance of airports in the overall transport network was characterized through four different local measures. Different outcome rankings using the four measures illustrate the fact that importance can be viewed through a number of lenses. By total strength of links (links weighted by traffic), it is not surprising to find Atlanta Hartsfield–Jackson ranked first followed by Chicago O'Hare. By the definition of eigenvector centrality, which accounts for weighted links of subject nodes and that of nearest neighbors, the most important designation swaps between the two. Measures that do not account for weighted links can uncover surprising findings, such as the importance of the Anchorage airport which connects the separated clusters of the continental U.S. and the large number of active airports in Alaska. Successful construction of a regression model to predict delayed operations exemplified the finding that these local measures can be meaningful and efficient indicators of network operational performance.

Overall, the merits of the network-theoretic approach appear to outweigh the limitations, especially from the perspective of obtaining useful knowledge with minimal modeling complexity by drawing inferences on network connectivity performance from topology structure. Continued work to embed the approach in a larger framework for modeling in support of technology and policy decision making is recommended, especially to foster connectivity in air transportation networks that remains locally and globally robust in response to failures, shifts in demand, or other perturbations.

Appendix: Temporary Annex

Local statistical measures for the 35 U.S. airports included in the FAA's Operational Evolution Partnership are provided here.

Airport	Degree	Airport	Traffic Volume	Airport	Eigenvector Centrality	Airport	Clustering Coefficient	Airport	Betweenness Centrality
MSP	236	ATL	863,083	ORD	0.3199	SAN	0.5003	SEA	0.1125
DEN	222	ORD	853,908	ATL	0.2924	TPA	0.4758	MSP	0.0921
DFW	221	DFW	717,135	DFW	0.2612	DCA	0.4696	DEN	0.0764
ATL	218	DEN	509,023	LAX	0.2095	FLL	0.4611	PHX	0.0502
ORD	198	LAX	484,516	DEN	0.1941	CLE	0.4584	ORD	0.0386
LAS	193	DTW	459,587	PHX	0.1794	JFK	0.4492	DFW	0.0382
PHX	185	MSP	457,462	MSP	0.1755	BWI	0.4353	ATL	0.0307
DTW	181	CVG	439,663	LGA	0.1722	HNL	0.4347	LAS	0.0278
MEM	177	PHX	424,619	DTW	0.172	MDW	0.4287	HNL	0.0247
SLC	174	IAH	407,819	LAS	0.17	SFO	0.4175	PIT	0.0246
PIT	174	CLT	374,228	PHL	0.1657	LGA	0.4156	PDX	0.0214
CVG	173	PHL	367,648	EWR	0.1651	BOS	0.395	DTW	0.0207
EWR	170	LGA	362,082	BOS	0.1598	MCO	0.386	IAH	0.0191
PHL	168	IAD	356,867	IAD	0.1566	IAD	0.3846	SLC	0.0189
IAD	168	LAS	349,916	IAH	0.1546	CVG	0.3813	LAX	0.0184
MIA	166	BOS	326,444	CVG	0.1488	PDX	0.3793	STL	0.0166
CLT	165	EWR	325,216	DCA	0.1363	IAH	0.3786	BOS	0.0158
IAH	164	SEA	301,933	MCO	0.1334	EWR	0.3779	EWR	0.0138
MCO	163	SLC	283,383	CLT	0.1321	LAX	0.3679	MIA	0.0128
STL	160	PIT	277,446	SFO	0.124	CLT	0.3654	CLT	0.011
LAX	149	MCO	271,159	STL	0.1205	PHL	0.3623	MEM	0.0108
BWI	149	SFO	264,995	SEA	0.1139	STL	0.3601	SFO	0.0104
CLE	148	DCA	256,459	BWI	0.1121	DTW	0.344	IAD	0.01
SEA	146	STL	244,840	PIT	0.1103	MIA	0.3314	LGA	0.0097
LGA	146	MDW	239,646	CLE	0.1102	PIT	0.33	JFK	0.0093
BOS	145	BWI	234,286	SAN	0.1068	MEM	0.3234	PHL	0.0078
MDW	143	CLE	220,996	MDW	0.1057	ORD	0.3205	CVG	0.0059
FLL	132	MEM	198,455	SLC	0.1002	SEA	0.2982	MCO	0.0058
JFK	131	JFK	188,410	FLL	0.0935	SLC	0.2933	FLL	0.0054
SFO	130	TPA	184,503	MIA	0.0909	LAS	0.2859	BWI	0.0049
DCA	127	SAN	177,198	TPA	0.0896	ATL	0.2846	SAN	0.0036
TPA	126	PDX	170,200	JFK	0.087	PHX	0.2707	MDW	0.0035
PDX	107	FLL	169,334	MEM	0.0843	DFW	0.2541	DCA	0.0031
SAN	106	MIA	157,641	PDX	0.0706	MSP	0.229	CLE	0.0031
HNL	53	HNL	141,315	HNL	0.0274	DEN	0.2181	TPA	0.0027

Fig. A1 Local network measures for OEP 35.

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